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Contributions to discussion of

- Approaches 2/3 to slope stability analyses
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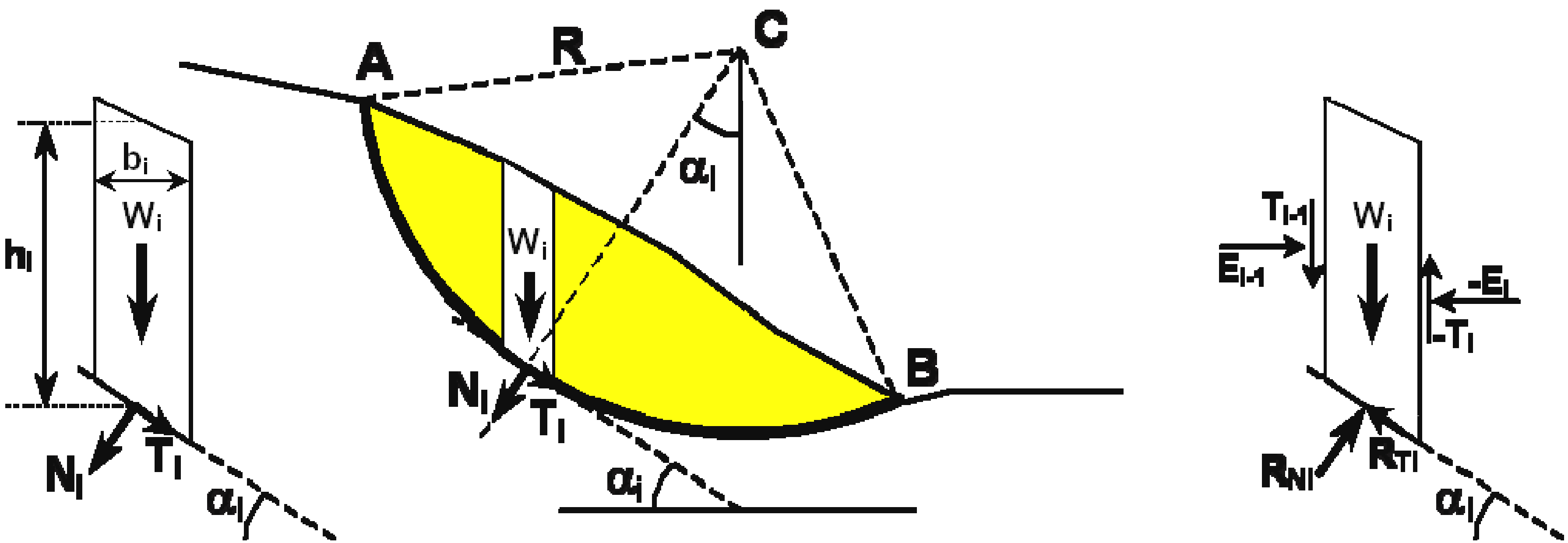
Slope stability analysis

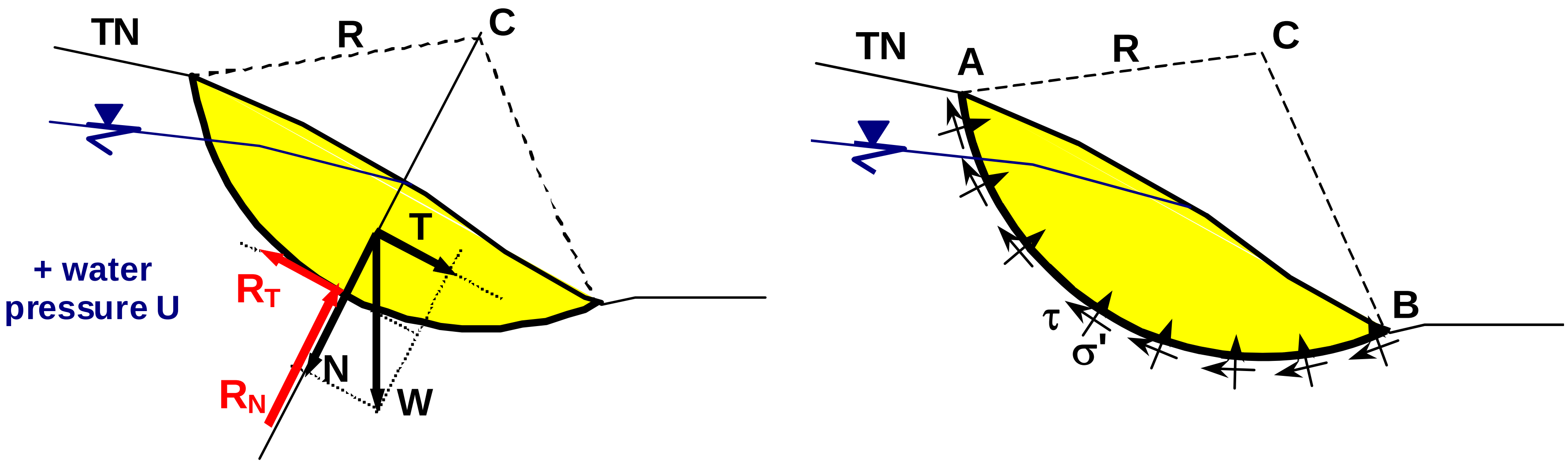
Methods used are

- Limit equilibrium of rigid blocks sliding on plane or curved (cylindrical surfaces)
- Limit analysis of blocks sliding on logarithmic spiral surface
- FE kinematic limit analysis
- FE elastoplastic deformation analysis.

The most used method is limit equilibrium.



The resolution needs a way to distribute the contact normal force on the limit of the block



One of the ways is the method of slices:

- Fellenius,
- Bishop (s)
- …

Fellenius method

$$F = \frac{\sum_i \left[c' b_i + \left(W_i \cos^2 \alpha_i - u_i b_i\right) \tan\varphi'\right] \frac{1}{\cos\alpha_i}}{\sum_i W_i \sin\alpha_i}$$

Bishop’s method

$$F = \frac{\sum_i \left[c'_i b_i + \left(W_i - u_i b_i\right) \tan\varphi'\right] \left[\left(1 + \frac{\tan\varphi' \tan\alpha_i}{F}\right) \cos\alpha_i\right]^{-1}}{\sum_i W_i \sin\alpha_i}$$

Stability verification (Fellenius, Approach 2)

$$\gamma_E \gamma_{Sd} \sum_i W_{ik} \, sin \, \alpha_i \leq \frac{1}{\gamma_R \gamma_{Rd}} \sum_i \left[c'_k \, b_i + \left(W_{ik} \, cos^2 \alpha_i - u_{ki} b_i \right) tan \, \varphi'_k \right] \frac{1}{cos \, \alpha_i}$$

Approach 2

- **γ_E = 1.35**
- **γ_{Sd} = 1,**
- **γ_R = 1,1,**
- **γ_{Rd} = 1.**

this can be rewritten as

$$\gamma_E \gamma_{Sd} \gamma_R \gamma_{Rd} \sum_i W_{ik} \, sin \, \alpha_i \leq \sum_i \left[c'_k \, b_i + \left(W_{ik} \, cos^2 \alpha_i - u_{ki} b_i \right) tan \, \varphi'_k \right] \frac{1}{cos \, \alpha_i}$$

Stability verification (Fellenius, Approach 3)

$$\gamma_E \gamma_{Sd} \sum_i \frac{W_{ik}}{\gamma_\gamma} \sin\alpha_i \le \frac{1}{\gamma_R \gamma_{Rd}} \sum_i \left[\frac{c'_k}{\gamma_{c'}} b_i + \left(\frac{W_{ik}}{\gamma_\gamma} \cos^2\alpha_i - u_{ki} b_i \right) \frac{\tan\varphi'_k}{\gamma_\varphi} \right] \frac{1}{\cos\alpha_i}$$

Approach 3

- **γ_E = 1**
- **γ_{Sd} = 1**
- **γ_γ = 1**
- **γ_R = 1**
- **γ_{Rd} = 1**
- **$\gamma_{c'}$ = 1.25**
- **$\gamma_{\varphi'}$ = 1.25**

since $\gamma_{c'} = \gamma_{\varphi'}$, this can be rewritten as

$$\gamma_{Sd} \sum_i \frac{W_{ik}}{\gamma_\gamma} \sin\alpha_i \le \frac{1}{\gamma_{Rd} \gamma_\varphi} \sum_i \left[c'_k b_i + \left(W_{ik} \cos^2\alpha_i - u_{ki} b_i \right) \tan\varphi'_k \right] \frac{1}{\cos\alpha_i}$$

and finally

$$\gamma_{Sd} \gamma_{Rd} \gamma_\varphi \sum_i \frac{W_{ik}}{\gamma_\gamma} \sin\alpha_i \le \sum_i \left[c'_k b_i + \left(W_{ik} \cos^2\alpha_i - u_{ki} b_i \right) \tan\varphi'_k \right] \frac{1}{\cos\alpha_i}$$

Both approaches are equivalent if:

$$\gamma_{Sd}\gamma_{Rd}\gamma_{\varphi'}\Big|_{approach\,3} = \gamma_{Sd}\gamma_{Rd}\gamma_{E}\gamma_{R}\Big|_{approach\,2}$$

This condition is verified if:

approach 3 :	$\gamma_{Sd} = 1$	$\gamma_{Sr} = 1$	$\gamma_{\varphi'} = 1{,}25$,	
approach 2 :	$\gamma_{Sd} = 1$	$\gamma_{Sr} = 1$	$\gamma_{E} = 1{,}35$	$\gamma_{R} = 1{,}1$

1.5 = 1 x 1 x 1.25 x 1.2 = 1 x 1 x 1.35 x 1.1 = 1.485

Suggestion to obtain the same number :

add a model factor on the resistance side in approach 3

γ_R **= 1.2**

Bishop's method

$$F = \frac{\sum_i \left[c' b_i + \left(W_i \cos^2 \alpha_i - u_i b_i\right) \tan \varphi'\right] \frac{1}{\cos \alpha_i}}{\sum_i W_i \sin \alpha_i}$$

$$F = \frac{\sum_i \left[c'_i b_i + \left(W_i \cos^2 \alpha_i - u_i b_i\right) \tan \varphi'\right] \frac{1}{\left[\left(1 + \frac{\tan \varphi' \tan \alpha_i}{F}\right) \cos \alpha_i\right]}}{\sum_i W_i \sin \alpha_i}$$

$$A = \left(1 + \frac{\tan \varphi' \tan \alpha_i}{F}\right)$$

has limited variations (2 to 2.2) and may be considered as a constant.

Conclusion: Bishop's formula has the same structure as Fellenius' one.
A model factor γ_R = 1.2 may be introduced in approach 3 to obtain a similar safety level.

Conclusion

Approaches 2 and 3 may be made equivalent for slope stability analyses